PRACTICE PAPER 01 FOR G.C.E A/L STUDENTS – 2024

CONDUCTED BY TAMIL STUDENTS OF FACULTY OF ENGINEERING

UNIVERSITRY OF MORATUWA

10 – COMBINED MATHEMATICS

ANSWERS (MARKING SCHEME)

Pure Mathematics

Part A

1. $\tan 6\theta = \tan(4\theta + 2\theta)$

$$= \frac{\tan 4\theta + \tan 2\theta}{1 - \tan 4\theta . \tan 2\theta} \quad \boxed{5}$$

$$= \frac{\frac{2\tan 2\theta}{1-\tan^2 2\theta} + \tan 2\theta}{1 - \frac{2\tan 2\theta}{1-\tan^2 2\theta}\tan 2\theta} \quad \boxed{5}$$

$$= \frac{2\tan 2\theta + \tan 2\theta - \tan^3 2\theta}{1 - tan^2 2\theta - 2\tan^2 2\theta}$$

$$= \frac{3\tan 2\theta - \tan^3 2\theta}{1 - 3\tan^2 2\theta} \quad \boxed{5}$$

$$= \frac{3p - p^3}{1 - 3p^2}$$

$$6\theta = \frac{3\pi}{4} \Rightarrow \theta = \frac{\pi}{8}$$

$$\tan\left(6.\frac{\pi}{8}\right) = \frac{3.\tan \pi/4 \quad -\tan^3 \pi/4}{1 - 3\tan^2 \pi/4} \quad \boxed{5}$$

$$\tan\left(\frac{3\pi}{4}\right) = \frac{3-1}{1-3}$$

$$= -1 \quad \boxed{5}$$

2. $\cos\theta(2\sin\theta - \tan^2\theta . \operatorname{cosec}\theta) = \sec^2\theta . \cos 2\theta$

$$\cos\theta\left(2\sin\theta - \frac{\sin^2\theta}{\cos^2\theta} \times \frac{1}{\sin\theta}\right) = \frac{1}{\cos^2\theta}(\cos^2\theta - \sin^2\theta) \quad \boxed{5}$$

$$\cos\theta\left(2\sin\theta - \frac{\sin\theta}{\cos^2\theta}\right) = 1 - \frac{\sin^2\theta}{\cos^2\theta}$$

$$2\sin\theta . \cos^3\theta - \sin\theta . \cos\theta = \cos^2\theta - \sin^2\theta$$

$$\sin\theta . \cos\theta\,(2\cos^2\theta - 1) = \cos^2\theta - \sin^2\theta \cos 2\theta\,(\sin\theta . \cos\theta - 1) = 0 \quad \boxed{5}$$

$$\sin 2\theta \neq 2, \quad \text{so } \cos 2\theta = 0$$

$$\cos 2\theta = \cos\frac{\pi}{2} \quad \boxed{5}$$

$$2\theta = 2n\pi \pm \frac{\pi}{2} \;; \; n \epsilon Z^+ \quad \boxed{5}$$

$$\theta = n\pi \pm \frac{\pi}{4}, \quad \theta = \frac{\pi}{4}, \frac{3\pi}{4} \quad \boxed{5}$$

3. $\dfrac{1 - \cos[\sin^{-1}(\tan x)]}{\tan^2 x} = \dfrac{1}{1 + \sqrt{1 - \tan x}}$

assume that, $\sin^{-1}(\tan x) = \alpha, \left(0 < \alpha \leq \frac{\pi}{2}, \; 0 < \sin\alpha \leq 1\right)$

$$\sin\alpha = \tan x$$

$$\cos^2\alpha = 1 - \sin^2\alpha$$

$$\cos\alpha = \sqrt{1 - \tan^2 x}, \quad (\cos\alpha > 0) \quad \boxed{5}$$

$$\text{L.H.S} = \frac{1 - \cos\alpha}{\tan^2 x}$$

$$= \frac{1 - \sqrt{1 - \tan^2 x}}{\tan^2 x}$$

$$= \frac{\left(1 - \sqrt{1 - \tan^2 x}\right)\left(1 + \sqrt{1 - \tan^2 x}\right)}{\tan^2 x.\left(1 + \sqrt{1 - \tan^2 x}\right)} \quad \boxed{5}$$

$$= \frac{1 - (1 - \tan^2 x)}{\tan^2 x.\left(1 + \sqrt{1 - \tan^2 x}\right)}$$

$$= \frac{\tan^2 x}{\tan^2 x.\left(1+\sqrt{1-\tan^2 x}\right)}$$

$$= \frac{1}{1+\sqrt{1-\tan^2 x}}$$

$$\frac{1-\cos\left[\sin^{-1}(\tan x)\right]}{\tan^2 x} = 1$$

$$\frac{1}{1+\sqrt{1-\tan^2 x}} = 1 \qquad \boxed{5}$$

$\tan x \rightarrow \sin 2y$

$$\frac{1}{1+\sqrt{1-\sin^2 2y}} = 1$$

$$\frac{1}{1+\cos 2y} = 1$$

$$\frac{1}{2cos^2 y} = 1 \qquad \boxed{5}$$

$\cos^2 y = \frac{1}{2}, \quad \cos y = \pm\frac{1}{\sqrt{2}}$

but, $0 < y \leq \frac{\pi}{4}$ because, $(0 < \sin 2y \leq 1,)$

$\cos y = \frac{1}{\sqrt{2}}, \quad \sin y = \frac{1}{\sqrt{2}}$

$\tan x = 2\sin y\cos y$

$$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1 \qquad \boxed{5}$$

4. $$\frac{\cot\theta - 1 + \operatorname{cosec}\theta}{\cot\theta + 1 - \operatorname{cosec}\theta} = \frac{1}{\operatorname{cosec}\theta - \cot\theta}$$

$$\text{L.H.S} = \frac{\cot\theta - 1 + \operatorname{cosec}\theta}{\cot\theta + 1 - \operatorname{cosec}\theta} \times \frac{(\cot\theta - \operatorname{cosec}\theta)}{(\cot\theta - \operatorname{cosec}\theta)} \qquad \boxed{10}$$

$$= \frac{\cot^2 x - \operatorname{cosec}^2 x - (\cot\theta - \operatorname{cosec}\theta)}{(\cot\theta + 1 - \operatorname{cosec}\theta)(\cot\theta - \operatorname{cosec}\theta)} \qquad \boxed{5}$$

$$= \frac{-1-(\cot\theta - \operatorname{cosec}\theta)}{(\cot\theta + 1 - \operatorname{cosec}\theta)(\cot\theta - \operatorname{cosec}\theta)} \qquad \boxed{5}$$

$$= \frac{-(\cot\theta + 1 - \operatorname{cosec}\theta)}{(\cot\theta + 1 - \operatorname{cosec}\theta)(\cot\theta - \operatorname{cosec}\theta)}$$

$$= \frac{1}{\operatorname{cosec}\theta - \cot\theta} \qquad \boxed{5}$$

5. $$\cos^{-1}\left(\frac{y^2-1}{y^2+1}\right) + \sin^{-1}\left(\frac{-2y}{1+y^2}\right) - \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \frac{\pi}{13} \qquad \boxed{5}$$

assume that, $y \rightarrow \tan\theta$

$$\cos^{-1}\left(\frac{\tan^2\theta - 1}{\tan^2\theta + 1}\right) + \sin^{-1}\left(\frac{-2\tan\theta}{1+\tan^2\theta}\right) - \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = \frac{\pi}{13}$$

$$\cos^{-1}(-\cos 2\theta) + \sin^{-1}(-\sin 2\theta) - \tan^{-1}(\tan 2\theta) = \frac{\pi}{13} \qquad \boxed{10}$$

$$\pi - 2\theta - 2\theta - 2\theta = \frac{\pi}{13}$$

$$6\theta = \frac{12}{13}\pi \qquad \boxed{5}$$

$$\theta = \frac{2}{13}\pi$$

$$y = \tan\left(\frac{2}{13}\pi\right) \qquad \boxed{5}$$

6. $a = b.\text{Cos } C + c.\text{Cos } B$

$b.\text{Cos } C = a - c.\text{Cos } B$

$\text{Cos } C = \frac{a}{b} - \frac{c}{b}\text{Cos } B$ $\boxed{5}$

$= \alpha - \beta \text{ Cos } B$

$\alpha + \beta = 2$

$\frac{a}{b} + \frac{c}{b} = 2$ $\boxed{5}$

$a + c = 2b$

So it lies in an arithmetic progression.

Here, $\alpha = \frac{a}{b}$ and $\beta = \frac{c}{b}$

$\beta - \alpha = \frac{4}{5}$

$\frac{c-a}{b} = \frac{4}{5}$ $\boxed{5}$

$c - a = 4$

Common difference = 2

$a = 3,\ b = 5,\ c = 7$

$\text{Cos } C = \frac{3}{5} - \frac{7}{5} \times \frac{11}{14}$ $\boxed{5}$

$= \frac{6-11}{10}$

$= -\frac{1}{2}$

$C = \frac{2\pi}{3},\ 0 < C < \pi$ $\boxed{5}$

So it is an Obtuse angled Triangle.

7. $\frac{(n+2)\pi}{3} = \alpha$

$(n + 2)\pi = 3\alpha \longrightarrow \text{(i)}$ $\boxed{5}$

$\text{(i)} \rightarrow (n + 2)\pi + \alpha = 3\alpha + \alpha$

$\tan 4\alpha = \tan \alpha$

and, $3\alpha - \alpha = (n + 2)\pi - \alpha$ $\boxed{5}$

$\tan 2\alpha = -\tan \alpha$

So, $\tan 4\alpha = -\tan 2\alpha$

$\tan 4\alpha + \tan 2\alpha = 0$

$\frac{2\tan 2\alpha}{1-\tan^2 2\alpha} = -\tan 2\alpha$ $\boxed{5}$

assume that, $\tan 2\alpha \rightarrow t$

$\frac{2t}{1-t^2} = -t$

$t^3 - t = 2t$ $\boxed{5}$

$t(t^2 - 3) = 0$

$\alpha > 0,\ \tan 2\alpha > 0 \longrightarrow t > 0$

So, $t^2 = 3$

$t = \pm\sqrt{3},$

$t = \sqrt{3} = \tan 2\alpha$ $\boxed{5}$

$2\alpha = \frac{\pi}{3} \longrightarrow \alpha = \frac{\pi}{6}$

Part B

8. (a) $\sin(A+B) = \sin A\cos B + \sin B\cos A$ $\boxed{5}$

$A \longrightarrow A\ , B \longrightarrow (-B)$

$\sin(A-B) = \sin(A)\cos(-B) + \cos(A)\sin(-B)$

$= \sin A\cos B - \cos A\sin B$

$A = \frac{\pi}{3}, B = \frac{\pi}{4}$ $\boxed{5}$

$\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{(\sqrt{3}-1)^2}}{2\sqrt{2}} = \frac{\sqrt{2-\sqrt{3}}}{2}$ $\boxed{5}$

$\sin(A-B) = \sin A\cos B - \sin B\cos A$

$A \longrightarrow \frac{\pi}{2}\ , B \longrightarrow A$

$\sin\left(\frac{\pi}{2} - A\right) = \sin\left(\frac{\pi}{2}\right)\cos A - \cos\left(\frac{\pi}{2}\right)\sin A$ $\boxed{5}$

$= \cos A$

$L.H.S = \sin(A+B)\sin(A-B)$

$= (\sin A\cos B + \sin B\cos A) \times (\sin A\cos B - \sin B\cos A)$ $\boxed{5}$

$= \sin^2 A\cos^2 B - \sin^2 B\cos^2 A$

$= \sin^2 A\,(1 - \sin^2 B) - \sin^2 B\,(1 - \sin^2 A)$ $\boxed{5}$

$= \sin^2 A - \sin^2 B$

$= R.H.S$

$L.H.S = \cos 10^\circ . \cos 30^\circ . \cos 50^\circ . \cos 70^\circ$ $\boxed{5}$

$= \sin 80^\circ . \sin 60^\circ . \sin 40^\circ . \sin 20^\circ$

$= \frac{\sqrt{3}}{2}\sin 20^\circ\,[\sin(60^\circ + 20^\circ)\sin(60^\circ - 20^\circ)]$

$= \frac{\sqrt{3}}{2}\sin 20^\circ\,(\sin^2 60^\circ - \sin^2 20^\circ)$ $\boxed{5}$

$= \frac{\sqrt{3}}{2} \times \sin 20^\circ\left(\frac{3}{4} - \sin^2 20^\circ\right)$

$= \frac{\sqrt{3}}{2} \times \frac{(3\sin 20^\circ - 4\sin^3 20^\circ)}{4}$ $\boxed{5}$

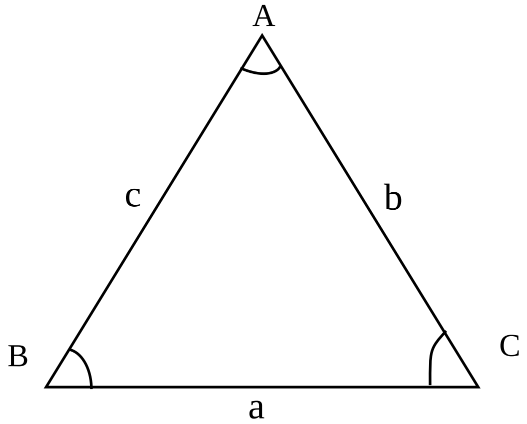
$= \frac{\sqrt{3}}{8}\sin 60^\circ$ $\boxed{5}$

$= \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2}$

$= \frac{3}{16}$

$= R.H.S$

(b)

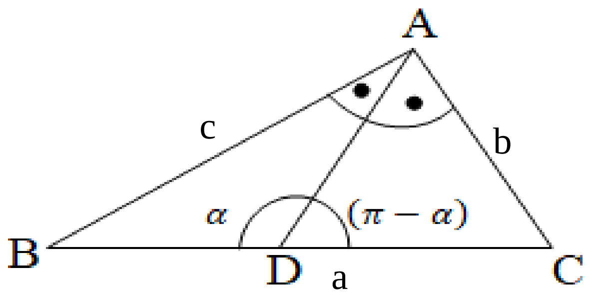


Sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad \boxed{5}$$

Cosine rule:

$$\cos A = \frac{b^2+c^2-a^2}{2bc} \qquad \boxed{5}$$



Δ ABD Sine rule:

$$\frac{BD}{\sin\left(\frac{A}{2}\right)} = \frac{c}{\sin\alpha} \longrightarrow (1) \qquad \boxed{5}$$

Δ ADC Sine rule:

$$\frac{DC}{\sin\left(\frac{A}{2}\right)} = \frac{b}{\sin(\pi-\alpha)}$$

$$\frac{DC}{\sin\left(\frac{A}{2}\right)} = \frac{b}{\sin\alpha} \longrightarrow (2) \qquad \boxed{5}$$

$$(1) + (2) \longrightarrow \frac{BD+DC}{\sin\left(\frac{A}{2}\right)} = \frac{c+b}{\sin\alpha} \qquad \boxed{5}$$

$(BD + DC = BC = a)$

$$\frac{a}{b+c} = \frac{\sin\left(\frac{A}{2}\right)}{\sin\alpha} \qquad \boxed{5}$$

$$\frac{a}{b+c} = \frac{BD}{c}$$

$$BD = \frac{ca}{b+c}$$

Δ ABC Sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} \longrightarrow (3) \qquad \boxed{5}$$

Δ ABD Sine rule:

$$\frac{AD}{\sin B} = \frac{BD}{\sin\left(\frac{A}{2}\right)} \longrightarrow (4) \qquad \boxed{5}$$

$(3),(4) \longrightarrow \dfrac{a}{\sin A} = \dfrac{b}{AD \sin\left(\frac{A}{2}\right)} \times BD$

$AD \cdot \dfrac{a}{2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)} = \dfrac{b}{\sin\left(\frac{A}{2}\right)} \cdot \dfrac{ca}{b+c}$ $\boxed{5}$

$AD = \dfrac{2bc\cos\left(\frac{A}{2}\right)}{b+c}$

<u>Δ ABC cosine rule:</u>

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$
$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$
$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

$$\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca} \quad \boxed{5}$$
$$\frac{2}{a}\left(\frac{c^2+b^2-a^2}{2bc}\right) + \frac{1}{b}\left(\frac{c^2+a^2-b^2}{2ac}\right) + \frac{2}{c}\left(\frac{b^2+a^2-c^2}{2ab}\right) = \frac{a}{bc} + \frac{b}{ca}$$

$$2(c^2+b^2-a^2) + (c^2+a^2-b^2) + 2(b^2+a^2-c^2) = a^2+b^2$$
$$a^2 = b^2+c^2$$

This agrees with pythagoras theorem so this is a right angled triangle. $\boxed{5}$

(c) Assume that, $\sin^{-1} x = \theta$

$x = \sin\theta$ $\boxed{5}$

$x = \cos\left(\dfrac{\pi}{2} - \theta\right)$

So, $\cos^{-1} x = \dfrac{\pi}{2} - \theta$ $\boxed{5}$

$= \dfrac{\pi}{2} - \sin^{-1} x$

$\cos^{-1} x + \sin^{-1} x = \dfrac{\pi}{2}$

$\cos(2\cos^{-1} x + \sin^{-1} x) = \dfrac{-2\sqrt{6}}{5}$

$\cos[\cos^{-1} x + (\cos^{-1} x + \sin^{-1} x)] = \dfrac{-2\sqrt{6}}{5}$

$\cos\left(\cos^{-1} x + \frac{\pi}{2}\right) = \frac{-2\sqrt{6}}{5}$ $\boxed{5}$

$-\sin(\cos^{-1} x) = \dfrac{-2\sqrt{6}}{5}$

$\sin(\cos^{-1} x) = \dfrac{2\sqrt{6}}{5} \longrightarrow R1$ $\boxed{5}$

$\cos^{-1} x = a \rightarrow \cos a = x \ \ (0 \le \cos^{-1} x \le \pi \ , 0 \le a)$

$\sin^2 a + \cos^2 a = 1$

$\sin^2 a = 1 - x^2$

$\sin a = \sqrt{1-x^2} \ \ (0 \le \sin a)$ $\boxed{5}$

$a = \sin^{-1}\left(\sqrt{1-x^2}\right)$

$\cos^{-1} x = \sin^{-1}\left(\sqrt{1-x^2}\right)$

$\sin(\cos^{-1} x) = \sqrt{1-x^2}$ $\boxed{5}$

$R1 \rightarrow \sin(\cos^{-1} x) = \frac{2\sqrt{6}}{5}$

$\sqrt{1-x^2} = \frac{2\sqrt{6}}{5}$ $\boxed{5}$

$1 - x^2 = \frac{24}{25}$

$x^2 = \frac{1}{25}$

$x = \frac{1}{5}\ \ (x > 0)$ $\boxed{5}$

Applied Mathematics
Part A

1. By the conservation of energy,

$$\frac{1}{2}m\left(\sqrt{\frac{ga}{2}}\right)^2 + mga = mga\cos\theta + \frac{1}{2}mV^2 \quad \boxed{10}$$

$$\frac{mga}{4} + mga = mga\cos\theta + \frac{1}{2}mV^2$$

$$\swarrow F = ma$$

$$R + mg\cos\theta = \frac{mV^2}{a} \quad \boxed{5}$$

$$R + mg\cos\theta = mg\left(\frac{5}{2} - 2\cos\theta\right)$$

$$R = mg\left(\frac{5}{2} - 3\cos\theta\right)$$

$$R = mg$$

$$\cancel{mg} = \cancel{mg}\left(\frac{5}{2} - 3\cos\theta\right)$$

$$3\cos\theta = \frac{3}{2}$$

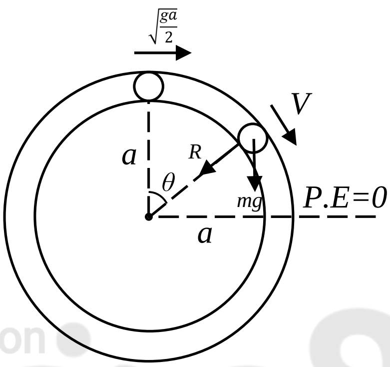
$$\cos\theta = \frac{1}{2} \quad \boxed{5}$$

$$\theta = \frac{\pi}{3}$$

$$v \longrightarrow u \ , \theta = \frac{\pi}{3}$$

$$u^2 = \left(\frac{5}{2} - 1\right)ga$$

$$u = \sqrt{\frac{3ga}{2}} \quad \boxed{5}$$



2. $F = ma$

$$\uparrow T\cos\theta = mg \quad \boxed{5}$$

$$\leftarrow T\sin\theta = mr\omega^2 \quad \boxed{5}$$

$$\tan\theta = \frac{r\omega^2}{g} \quad \boxed{5}$$

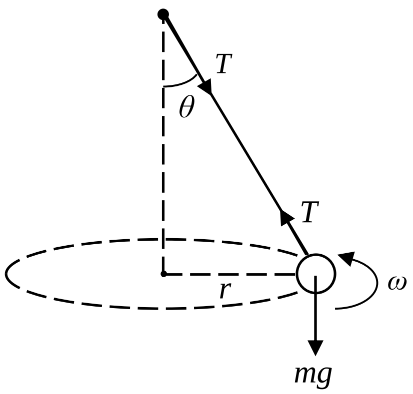
$$\frac{\pi}{4} < \theta$$

$$\tan\frac{\pi}{4} < \tan\theta \quad \boxed{5}$$

$$1 < \frac{r\omega^2}{g} \quad \boxed{5}$$

$$\frac{g}{r} < \omega^2$$

$$\omega > \sqrt{\frac{g}{r}}$$



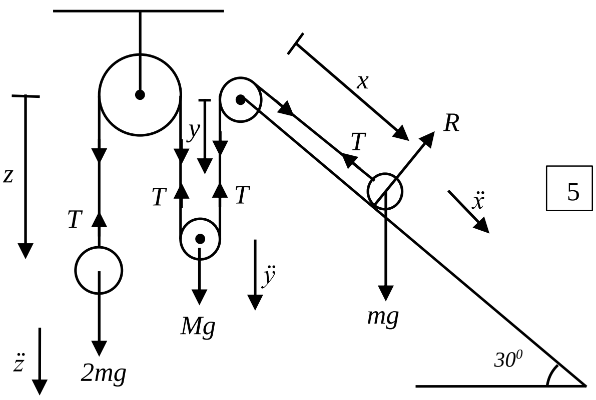
3. $\ddot{x} + 2\ddot{y} + \ddot{z} = 0$ $\boxed{5}$

$F = ma$

$\searrow mg\sin 30 - T = m\ddot{x}$

$\downarrow Mg - 2T = M\ddot{y}$

$\downarrow 2mg - T = 2m\ddot{z}$ $\boxed{10}$



$\boxed{5}$

$\dfrac{3mg}{2} = 2m\ddot{z} - m\ddot{x}$

$3g = 4\ddot{z} - 2\ddot{x}$

$3g = \dfrac{6g}{5} - 2\ddot{x}$

$2\ddot{x} = -\dfrac{9g}{5}$

$\ddot{x} = -\dfrac{9g}{10}$

$\ddot{y} = \dfrac{3g}{10}$

$mg - 2T = 2m\ddot{x}$

$Mg - mg = M\ddot{y} - 2m\ddot{x}$

$Mg - mg = \dfrac{3Mg}{10} + \dfrac{18mg}{10}$

$10M - 10m = 3M + 18m$

$M = 4m$ $\boxed{5}$

4. Length of string

$x + 2y =$ constant $\boxed{5}$

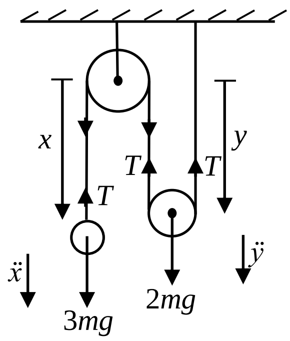
$\dot{x} + 2\dot{y} = 0$

$\ddot{x} + 2\ddot{y} = 0$

$F = ma$

$\downarrow 3mg - T = 3m\ddot{x}$ $\boxed{5}$

$\downarrow 2mg - 2T = 2m\ddot{y}$ $\boxed{5}$



$\boxed{5}$

$mg - T = m\ddot{y}$

$3mg - T - mg + T = 3m\ddot{x} - m\ddot{y}$

$2mg = 3m\ddot{x} - m\left(-\dfrac{\ddot{x}}{2}\right)$

$2g = 3\ddot{x} + \dfrac{\ddot{x}}{2}$

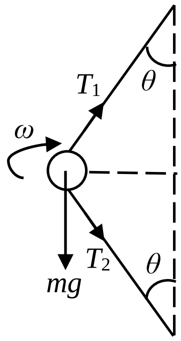
$4g = 7\ddot{x}$

$\ddot{x} = \dfrac{4}{7}g$ $\boxed{5}$

5. $\uparrow T_1 \cos\theta = T_2 \cos\theta + mg$ $\boxed{5}$

$(T_1 - T_2)\cos\theta = mg$

$\rightarrow T_1 \sin\theta + T_2 \sin\theta = m(l\sin\theta)\omega^2$ $\boxed{5}$



$\boxed{5}$

$(T_1 + T_2)\sin\theta = ml\sin\theta \times \omega^2$

$T_1 + T_2 = ml\omega^2$

$T_1 - T_2 = mg\sec\theta$

$2T_1 = ml\omega^2 + mg\sec\theta$

$T_1 = \dfrac{m}{2}(l\omega^2 + g\sec\theta)$ $\boxed{5}$

$T_2 = \dfrac{m}{2}(l\omega^2 - g\sec\theta)$ $\boxed{5}$

6. $\downarrow 5mg - T = 5m\ddot{x}$ $\boxed{5}$

$\downarrow 2mg - T_1 = 2m\ddot{y}$ $\boxed{5}$

$\downarrow mg + T_1 - 2T = m\ddot{y}$ $\boxed{5}$

$\overleftarrow{F=ma}$

$T = 3m(-\ddot{z})$

$F = ma$ $\boxed{5}$

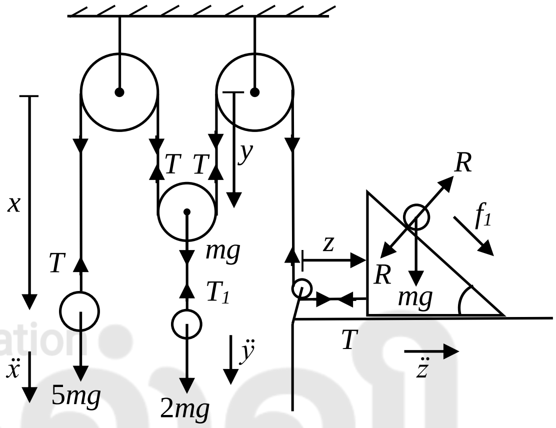
$mg\sin\theta = m(\ddot{z}\cos\theta + g\sin\theta)$

Length of string

$x + 2y + z =$ constant

$\dot{x} + 2\dot{y} + \dot{z} = 0$

$\ddot{x} + 2\ddot{y} + \ddot{z} = 0$



$\boxed{5}$

7. By the conservation of energy,

$\frac{1}{2}m\left(\sqrt{kga}\right)^2 - mga = \frac{1}{2}mV^2 - mga\cos\alpha$ $\boxed{5}$

$v^2 = kga - 2ga(1 - cos\,\alpha)$

$\alpha = \pi \qquad v = u$

$u^2 = kga - 4ga$

$\downarrow F = ma$ $\boxed{5}$

$mg = \dfrac{mu^2}{a}$

$ga = kga - 4ga$

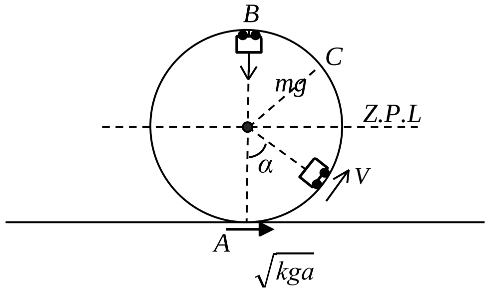
$k = 5$ $\boxed{5}$

$\alpha = \pi - \theta \qquad v = w$

$w^2 = 5ga - 2ga(1 - \cos(\pi - \theta))$ $\boxed{5}$

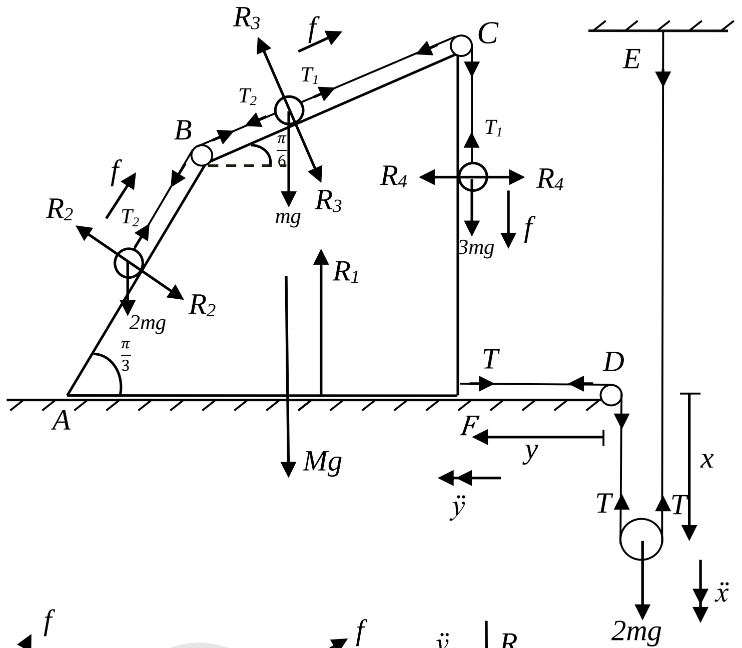
$w^2 = 3ga - 2ga\cos\theta$

$w^2 = ga(3 - 2\cos\theta)$ $\boxed{5}$

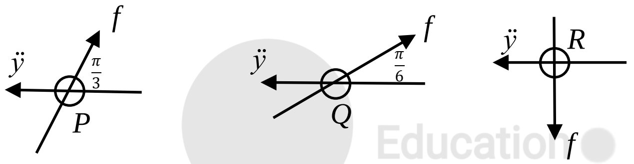


Part B

(a)



10	Force
10	Acceleration



$\overleftarrow{F=ma}$ for system

$$-T = M\ddot{y} + 2m\left(\ddot{y} - f\cos\frac{\pi}{3}\right) + m\left(\ddot{y} - f\cos\frac{\pi}{6}\right) + 3m\ddot{y} \qquad \boxed{15}$$

for particle P

$$\swarrow \quad 2mg\sin\frac{\pi}{3} - T_2 = 2m\left(\ddot{y}\cos\frac{\pi}{3} - f\right) \qquad \boxed{10}$$

for particle Q

$$\nearrow \quad T_1 - T_2 - mg\sin\frac{\pi}{6} = m\left(f - \ddot{y}\cos\frac{\pi}{6}\right) \qquad \boxed{10}$$

for particle R

$$\downarrow \quad 3mg - T_1 = 3mf \qquad \boxed{5}$$

Pulley S

$$\downarrow \quad 2mg - 2T = 2m\ddot{x} \qquad \boxed{5}$$

Length of string

$2x + y =$ Constant

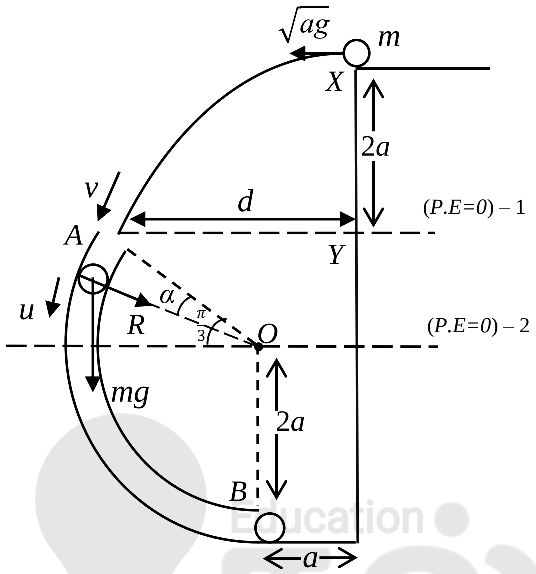
$2\ddot{x} + \ddot{y} = 0$

for particle P, relative to wedge

$\nearrow$ $S = ut + \frac{1}{2}at^2$

$a = 0 + \frac{1}{2}ft^2$ $\boxed{5}$

(b)



$X \longrightarrow A$

$S = ut + \frac{1}{2}at^2$

$2a = 0 + \frac{1}{2}gt^2$

$t^2 = \frac{4a}{g}$

$t = \pm 2\sqrt{\frac{a}{g}}$

But $t > 0$

$\therefore t = 2\sqrt{\frac{a}{g}}$ $\boxed{5}$

d be the horizontal range attained by particle P in time t.

$s = ut$

$d = \sqrt{ag} \times 2\sqrt{\frac{a}{g}}$ $\boxed{5}$

$d = 2a$

But $AX = 2a$

$\therefore d = AY$

Therefore particle P goes through end A. $\boxed{5}$

By the law of conservation of energy to particle P at point A ,

$$\frac{1}{2}m\left(\sqrt{ag}\right)^2 + mg(2a) = \frac{1}{2}mV^2$$
$$V^2 = ag + 4ag$$
$$V^2 = 5ag$$
$$V = \sqrt{5ag} \quad \boxed{5}$$

By the conservation of energy,

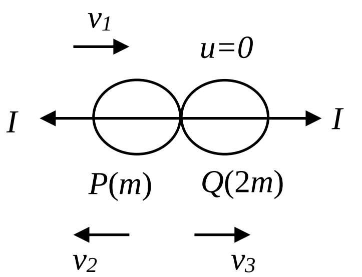
$$\frac{1}{2}mV^2 + mg(2a\cos 30) = \frac{1}{2}mu^2 + mg \times 2a\cos(30+\alpha) \quad \boxed{10}$$
$$\frac{1}{2}m(5ag) + mg(2a)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{2}mu^2 + mg(2a)\left(\frac{\sqrt{3}}{2}\cos\alpha - \frac{1}{2}\sin\alpha\right)$$
$$5ag + 2\sqrt{3}ag = u^2 + 2ag\left(\sqrt{3}\cos\alpha - \sin\alpha\right)$$
$$u^2 = ag\left[5 + 2\sqrt{3} - 2\left(\sqrt{3}\cos\alpha - \sin\alpha\right)\right] \quad \boxed{5}$$

$\nwarrow F = ma$

$$R + mg\cos(30+\alpha) = m\frac{u^2}{2a} \quad \boxed{5}$$
$$R = \frac{mg}{2}\left[5 + 2\sqrt{3}\right]$$
$$R + mg\cos(30+\alpha) = \frac{5mg}{2} + \sqrt{3}mg - 2mg\cos(\alpha+30)$$
$$R = \frac{5mg}{2} + \sqrt{3}mg - 3mg\left(\frac{\sqrt{3}}{2}\cos\alpha - \frac{\sin\alpha}{2}\right)$$
$$R = \frac{mg}{2}\left[5 + 2\sqrt{3} - 3\left(\sqrt{3}\cos\alpha - \sin\alpha\right)\right] \quad \boxed{5}$$

let $u = v_1$ when $\alpha = 150$,

$$v_1{}^2 = ag\left[5 + 2\sqrt{3} - 2\left(\sqrt{3} \times \left(-\frac{\sqrt{3}}{2}\right) - \frac{1}{2}\right)\right]$$
$$v_1{}^2 = ag\left(9 + 2\sqrt{3}\right) \quad \boxed{5}$$



$\overrightarrow{I=\Delta mv}$ for system

$$0 = 2mv_3 - mv_2 - mv_1$$
$$v_1 = 2v_3 - v_2 \quad \boxed{5}$$

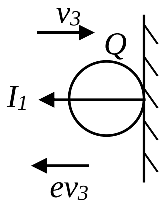
$v_1 e = v_2 + v_3$ $\boxed{5}$

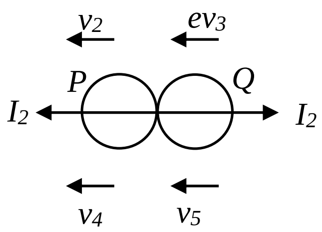
$v_2 = \left(\frac{2e-1}{3}\right) v_1 \quad , \quad v_3 = \left(\frac{1+e}{3}\right) v_1$

If $= \frac{1}{2}$,

$v_2 = 0$

$v_3 = \frac{v_1}{2}$





$\overleftarrow{I = \Delta mv}$

$0 = mv_4 + 2mv_5 - 2m\left(\frac{v_1}{4}\right)$ $\boxed{5}$

$\frac{m_1}{2} = v_4 + 2v_5$

$\left(e = \frac{1}{2}\right)$ Newton's experimental law,

$\left(\frac{v_1}{4}\right)\frac{1}{2} = v_4 - v_5$ $\boxed{5}$

$\frac{v_1}{4} = 2v_4 - 2v_5$

$\frac{3v_1}{4} = 3v_4$

$v_4 = \frac{v_1}{4}$ $\boxed{5}$

By the conservation of energy,

$\frac{1}{2}mv_4{}^2 = \frac{1}{2}m(0) + mgh$

$h = \frac{v_4{}^2}{2g}$

$h = \frac{v_1{}^2}{32g} = \frac{1}{32}(a)(9 + 2\sqrt{3})$

$h = \left(\frac{9+2\sqrt{3}}{32}\right)a$ $\boxed{5}$