Provincial Department of Education - Northern Province

Combined Mathematics

Grade:12 (2024) | **10** | **E** | **Three hours**
Additional Reading Time : 10 minutes

Index No. | | | | | | |

Instructions

- This question paper consists of two parts;

  Part A (questions 1-10) and **part B**(questions 11- 17).

Part - A

- Answer **all** questions. Answers should be written in the space provided on the questions paper. If additional space needed, you may use additional answer sheets.

Part - B

- Answer only 5 questions.
- After the allocated time hand over the paper to the supervisor with both parts attached together.
- **Only part B** of the paper is allowed to be taken out of the Examination Hall..

(10) Combined Mathematics		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
Total		

Final Marks

Part A

1. Let $f(x) = 2kx^2 + 2(k+4)x + 9 \,.\, (k \neq 0)$ x Find the range of k when $f(x)$ is positive for all real values of x.

2. Find the range of x, $\frac{2}{x-1} \geq \frac{1}{x+3}$

3. Resolve into partial fraction $\frac{x^2+1}{x^2(x-2)}$

4. Solve $\frac{1}{2}\log_3 x - \log_x 9 + \frac{3}{2} = 0$

5.Using suitable substitute for $\left(x+\frac{2}{x}-1\right)\left(x+\frac{2}{x}+4\right)=6$ then find the value of

x^2+5x+2, x^2-2x+2 .

6. Prove that $\frac{Sin^3x}{1+Cosx}+\frac{Cos^3x}{1-Sinx}=\sqrt{2}Cos\left(\frac{\pi}{4}-x\right)$

7. $2\underline{a}+\underline{b}, \underline{b}$ are perpendicular to each other . Given that $\underline{a}$ is unit vector and $|\underline{b}|=\sqrt{3}$ .Find the angle between $\underline{a}, \underline{b}$.

8. The position vectors of A *and* B with respect to O are $\underline{a}$ *and* $\underline{b}$ respectively .The point C lies on produced AB such that $AC=3BC$ .Find the position vector of C with respect to O in term $\underline{a}$ *and* $\underline{b}$.

9. the magnitude of the resultant of two forces P and Q acting at a point at an angle θ is P .When the direction of the force is reversed , the magnitude of the resultant is 2P show that $P^2 : Q^2 = 2:3$ and also determine the angle between P and Q .

10. There are three force act on a point are in equilibrium. The angle between the forces are $120°, 150° and\ 90°$. From these forces ,if the least force is 100N,find the magnitude of other two forces.

Part B

Answer five questions only.

11.(a) If $\alpha and \beta$ are the root of $ax^2 + bx + c = 0, (a \neq 0)$,

Show that $\alpha + \beta = \frac{-b}{a}$, $\alpha . \beta = \frac{c}{a}$.

let $f(x) = 2x^2 - 2(2-k)x + 1 - k$ .Here $k \in R$.

Show that the roots of $f(x) = 0$ are real and difference . If the roost of $f(x) = 0$ are γ, δ Find the quadratic equation whose roots are $\gamma^2 + 2, \delta^2 + 2$ interim k.

(b) Let $f(x) = 2x^3 + 9x^2 + 10x + 3$ Show that (2x + 1) is a factor of $f(x)$. write $f(x)$ as a product of three linear factors. Find the remain when $f(x)$ is divided by $2x^2 + 3x - 2$

12. (a) Solve the following equation

(i) $5x^3 + 31x^2 + 31x + 5 = 0$

(ii) $\sqrt{3x+1} - \sqrt{2-x} = \sqrt{2x-1}$

(iii) $2^{2x} - 3.2^{x+2} + 32 = 0$

(b) (i) Show that $\frac{1}{1+\log_a b + \log_a c} + \frac{1}{1+\log_b a + \log_b c} + \frac{1}{1+\log_c a + \log_c b} = 1$.

(ii) Solve $xy = 80$, $\log_{10} x - 2\log_{10} y = 1$

13.(a) Let take $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ then Show that $\tan\frac{\pi}{12} = 2 - \sqrt{3}$.

Deduced that $\tan\frac{13\pi}{12}$.

(b) Show that $\tan 3\theta - \tan\theta = \frac{2Sin\theta}{Cos3\theta}$.

Hence Show that , $\frac{Sin\theta}{Cos\ 3\theta} + \frac{Sin\ 3\theta}{Cos\ 9\theta} + \frac{Sin 9\theta}{Cos\ 27\theta} = \frac{1}{2}(\tan 27\theta - \tan\theta)$.

(c) Prove that $\frac{\cos 8\theta + \cos 6\theta}{\sin 8\theta + \sin 6\theta} = \cot 7\theta$.

14.(a) Let $\frac{3\pi}{2} < \theta < 2\pi$ if $Cot^2\theta = 4$,

then Show that $Sec\theta - Co\sec\theta = \frac{3\sqrt{5}}{2}$.

(b) Solve the equation $Cosx + Cos2x + Cos\,3x = Sinx + Sin\,2x + Sin\,3x$ in the rang $0 \leq x \leq \frac{\pi}{2}$,

(c) Show that $\sec^3 x + 2\sec^2 x.\tan x + \sec x.\tan^2 x = \frac{\cos x}{(1-\sin x)^2}$ Here $x \neq (2n+1)\frac{\pi}{2}$, $n \in Z$.

15.(a) The position vectors of A, B, C with respect to O are $12\boldsymbol{a}$, $4\boldsymbol{b}$, $12\boldsymbol{a}+8\boldsymbol{b}$ respectively. Here $\boldsymbol{a}$ and $\boldsymbol{b}$ are non parallel vectors .The point D lies on OA such that $OD:DA = 1:2$. The posit E lies on BD such that $BE:ED = 3:1$.The lines OE and AB are intersect at F.

I. Show that $\overrightarrow{OE} = 3\boldsymbol{a} + \boldsymbol{b}$.

II. If $\overrightarrow{BF} = \mu\overrightarrow{AB}$ find $\overrightarrow{BF}$in term $\mu, \boldsymbol{a}, \boldsymbol{b}$. write down another expression for $\overrightarrow{BF}$. then Show that The position vector of F with respect to O is $\overrightarrow{OF} = 6\boldsymbol{a} + 2\boldsymbol{b}$.

III. Show that D,F and C are collinear .

(b) Define the scalar product $\boldsymbol{a}.\boldsymbol{b}$.

OABC is a parallelogram . Here O is the origin. The position vectors of A *and* C with respect to O are $\boldsymbol{a} = i + \mu j$, $\boldsymbol{c} = 3i + j$ ($\mu > 0$)

I. If $\sqrt{2}OA = OC$ *then Show that* $\mu = 2$.

II. Find $\overrightarrow{OB}$, $\overrightarrow{AC}$ interims i , j , find the angle $\angle AOC$.

III. Find the area of parallelogram OABC.

16.(a)Two force $25N$ *and* $15N$ act on a particle .If the angle between the two forces is α ,where $\sin\alpha = \frac{4}{5}$ find the magnitude and direction of the resultant when ,

(i) α is acute .

(ii) α is obtuse .

(b) $ABCDEF$ is a regular hexagon , A coplanar system of force $3\sqrt{3}, 2, 4\sqrt{3}, 3$ *and* $3\sqrt{3}$ *Newton* act along the directions AB, AC, AD, AE *and* AF respectively . By finding the algebraic sum of the components of each fore in the system in the direction of AB and in a direction perpendicular to it find the magnitude and direction of the resultant.

17.(a) Three force P, Q *and* R act on a point are in equilibrium. If the angle between the force P *and* Q is twice the angle between the forces Q *and* R then using the Lami[1]s theorem show that $R^2 = P(P-Q)$.

(b) There are Three force act on a point are in equilibrium. If the angle between any two forces is $120°$,by using the law of triangle of force find the relationship among the magnitude of that forces .More over if the angles are $60°, 150°$ *and* $150°$ find the ratio among the magnitude of that forces.

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